LITTLE BLUE BOOK NO. 857
Edited by E. Haldeman-Julius

Arithmetic Self Taught

Part II

Lloyd E. Smith

LITTLE BLUE BOOK NO. 857
Edited by E. Haldeman-Julius

Arithmetic Self Taught

Part II

Lloyd E. Smith

HALDEMAN-JULIUS COMPANY
GIRARD, KANSAS

Copyright, 1925,
Haldeman-Julius Company

PRINTED IN THE UNITED STATES OF AMERICA

ARITHMETIC SELF TAUGHT
PART II

CONTENTS

Page

Foreword 5

I. Powers (Involution) 7

II. Roots (Evolution) 10

III. Interest 17

IV. Gain and Loss 21

V. Discounts 28

VI. Weights and Measures 32

VII. Area 39

VIII. Volume 49

IX. Practical Proportion 54

Answers to Exercises 59

Index (to Parts I and II)..........61

FOREWORD

Part I dealt with elementary operations in Arithmetic, gradually advancing toward the more difficult scope of Part II, in which more practical problems are discussed. One word of explanation is due every student of arithmetic concerning textbook problems. There is a certain idealization, always, of the conditions of a problem. The student must take it for granted that whatever the problem does not mention is to be ignored. The purpose of these problems is not to teach the student geometry, or physics, or geography, or what not—but to teach him the principles of arithmetic. Of course, facts have been followed pretty closely, in problems involving fixed facts such as the diameter of the earth and the speed of sound waves. Still, even if these facts were distorted, the point would not be in the exact physical figures, but in the process required to use them arithmetically.

Practical problems that the student will meet with in everyday life must be idealized to a certain extent also. Small elements of friction and variation can be ignored without a loss of accuracy sufficient to count, at least for all practical purposes. The average man is not interested in computing the dimensions of his floors to the thousandth of an inch—not even to the half inch. Square feet, in round numbers, is usually enough.

Once again the student is reminded that *A Handbook of Useful Tables* (Little Blue Book No. 835) contains much that will prove of value to him.

NOTE

I wish to thank Mr. H. S. Tallard and Mr. E. Newton Smith for carefully checking the problems of Parts I and II of this little arithmetic. In addition, I acknowledge my indebtedness to Mr. E. Newton Smith, my father, for preparing all the hand-drawn diagrams.

LLOYD E. SMITH.

ARITHMETIC SELF TAUGHT

PART II

I. POWERS (INVOLUTION)

A number multiplied by itself gives its *square*, or its *second power*. Thus, 2×2=4, so that 4 is the square of 2, or 4 is the second power of 2. Similarly, since 2×2×2=8, 8 is the *cube* or the *third power* of 2. Proceeding, 2×2×2×2=16, 16 is the fourth power of 2. Or, stating the matter in another way, if a product is made up of equal factors (16 is a product which is made up of four factors, all 2's, and so equal), this product is known as a *power* of one of these factors. The power is named to correspond with the number of equal factors in the product; hence, 4 is the *second* power, for it contains *two* equal factors.

The second power is usually called the *square* and the third power the *cube*, because if a square, for example, is drawn measuring 4 inches on a side, the area of that square will be 4×4=16 square inches; and, further, if on this square a cube is erected, measuring 4 inches on each dimension, the volume of that cube will be 4×4×4=64 cubic inches. (See Area and Volume.)

In higher mathematics the process of raising numbers to powers is termed *involution*. Instead of writing out the factors, which for large powers would be laborious, a shorter system of notation is used. To denote the second power of 4, for example, a small numeral 2 is placed above and to the right of the 4, a "superior" 2, as 4^2. This is read "four squared," or "four to the second power." Similarly, 3^4 indicates the fourth power of three:

$$3^4=3\times3\times3\times3=81$$

It should be noted that 81 is not only the fourth power of 3—it is also the square of 9. This little superior number is called an *exponent.* This exponent is usually read without the word "power," as 3^5="three fifth," meaning, of course, *three to the fifth power.*

Any number may be raised to any power by multiplying it out, using itself as a factor as many times as the exponent indicates. Thus, 13^3 and 1.3^4 may be obtained:

$$\begin{array}{r} 13\\ \times\ 13\\ \hline 39\\ 13\\ \hline 169\\ \times\ 13\\ \hline 507\\ 169\\ \hline 2197\end{array}$$

$$1.3^2=1.69$$
$$1.3^3=2.197$$

$$\begin{array}{r} 2.197\\ \times\ 1.3\\ \hline 6591\\ 2\,197\\ \hline 2.8561\end{array}$$

The third power of 13 is therefore 2197; and, since we found the square of 13 to be 169, the square of the decimal 1.3 will have the same figures, with two decimal places, and, similarly, the cube of 1.3 will be 2197 with three decimal places—so we find the fourth power of 1.3 to be 2.8561.

Any fraction may be raised to any power by raising both its numerator and its denominator to the power indicated. Thus:

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{3 \times 3 \times 3}{4 \times 4 \times 4} = \frac{27}{64}$$

The large parentheses around the entire fraction at the beginning show that the whole fraction is to be raised to the third power. If these parentheses were not used, a three against the numerator would indicate that the numerator only was to be raised to the third power. Either the parentheses must be used when it is desired to express a power of the whole fraction, or the exponent must be placed against both numerator and denominator. (As we learned under Fractions, Part I, as long as the same operation is performed—exclusive of addition and subtraction—on both numerator and denominator of any fraction, the value of the fraction remains unchanged.)

Exercises (correct answers at end of book):

(B1) Raise to the powers indicated; 4^5; 10^3; 1^6; 1.5^3; 21.52^2; $\left(\frac{2}{3}\right)^4$

NOTE.—An ordinary number, without an expressed exponent, is understood to have the exponent "1." Thus, 2^1 is equivalent to simply 2, for if the factor is taken only once, there is no product and consequently no power. The "first" power is therefore the number itself. Remember also that no matter how many times 1 is multiplied by itself, the product is always 1, so that any power of 1 is 1.

II. ROOTS (EVOLUTION)

It has been explained that if a product is made up of equal factors, this product is known as a power of one of these factors. Turning about, any one of these factors is said to be a *root* of the product. The process of finding the root of any number is known as extracting the root—it is the reverse of raising to a power, and is called, technically, *evolution* (the opposite of *involution*).

Since 4 is the square of 2, 2 is the *square root* of 4. Similarly, 9 is the square root of 81; 3 is the fourth root of 81. Since the student has learned the multiplication tables (Part I, Multiplication) up to 12, he knows the squares of all the numbers up to and including 12—and, conversely, he knows the square roots of the squares of these numbers. The root of a number may always be proved, for if it is known that 13 is the third root (cube root) of 2197, then 13^3 must equal 2197. If it is multiplied out and found to be true, then the root must be right.

As exponents were used to indicate the powers of numbers, so a special system is used to indicate their roots. A mark called the *radical sign* (the derivation of the word *radical* implied "root," from the Latin *radix*, root) is used, with a little figure placed over the Vshaped cavity of the radical sign, known as the *index* figure, to indicate the root desired:

$$\sqrt{4} = 2 \qquad \sqrt[5]{64} = 2$$

Usually the index 2 is omitted from the radical when only the square root is desired, the radical without an expressed index always signifying square root.

Before proceeding to the extraction of roots, it is necessary to point out that every number has what are known as *significant figures*. Beginning with the first digit at the left and ending with the last digit at the right (a cipher not being accounted a digit), these two digits and all digits or ciphers *between* them compose the *significant part* or the *significant figures* of the number. Thus, in 20,198,060, the significant part is 2019806; in 45,000,000 the significant part is 45. In decimals, the decimal point is not considered in determining the significant part, so that in 20500.0016, the significant part is 205000016. Entirely different *numbers* may have the same significant parts. Thus,

20,560
2,056
205,600
205,600,000
2.056
20.56
205.6
.2056
.00002056

all have the same significant part, 2056.

SQUARE ROOT.—In extracting square root, it should first be noted that the square root of any number with one or two figures will be one figure, but the square root of any number with three or four figures will be two figures. So, before extracting the square root of any number, it is desirable to know how many figures there will be in the root. This can be determined at once by marking off the number into *periods* of two figures each, proceeding from right to left from the decimal point (and, if a decimal, also from left to right from the decimal point). The number of periods will then show the number of figures in the root. Each period contains two figures, except perhaps the last figure on the left end of a whole number, which may form a period by itself. The last period of a decimal is completed, if necessary, by the addition of a cipher. By so marking off, we know that the square root of 2′05′66′72 contains 4 figures.

Of course, every number does not have an exact square root. A number which can be divided into exactly equal factors is said to be a *rational* number, and has rational factors—it is, in other words, a perfect power. Of all the numbers from 1 to 1000, there are only 42 perfect powers, and only 30 of these are perfect squares, and 9 are perfect cubes.

A number which cannot be divided exactly into equal factors is an imperfect power, and its factors, which are rational, are known as *surds*. The square root of an imperfect power is an interminable decimal—for example, the square root of 28 is an endless decimal, for 28 lies between the two perfect squares 25 (5^2) and 36 (6^2). Since the square root of 25 is 5, and the square root of 36 is 6, the square root of 28 is somewhere between 5 and 6, or is 5 plus an interminable decimal. (For all practical purposes, 5 with three decimal places would be accurate enough for the square root of 28. Or, a little better, it is wise to extract any square root to four or five *significant figures*, unless, of course, the root is a perfect one of less than that number.)

The precise method of finding the square root of a whole number, say 135,424, follows:

First, mark the number into periods of two figures each, counting from right to left:

13 ′ 54 ′ 24

It is usually customary to add the radical sign, to indicate the operation being performed upon the number. The root is then written at the right:

```
      ___________
     √ 13 ′ 54 ′ 24 ( 368
                      ____
        9
       __
66 )  4 54
____
      3 96
      ____
  728 )  58 24
         58 24
         _____
```

And 368 is the exact square root of 135,424.
Explanation: The first period (13) is examined to discover the largest exact square it contains. This is 9, for the next perfect square

(of 4) would be 16. Since the square of 3 is the largest square contained in 13, 3 is written down as the first figure of the root (at the right). The square of 3 is written under 13, and subtracted from it, leaving 4. The next period (54) is brought down and placed with the 4, making it 454. The part of the root thus far secured (3) is now doubled, and placed at the left of 454, to act as a trial divisor. But this divisor (6) is regarded as though it were of two figures (60)—or, perhaps better, the 6 is divided into the first figure or first two figures of the temporary dividend (45, for 6 won't go into 4), the idea being to discover whether it will go, say 7 times, if the 7 is used to complete the divisor, making it 67. But 67 will not go 7 times into 454, so 6 is the next figure of the root, and is placed at the upper right. A 6 is also placed in the divisor, making it 66, and 66 is now multiplied by this second figure of the root (6), and the product (396) is placed under 454 and subtracted from it. This leaves 58 and the next and last period (24) is brought down to make a new dividend (5824). The part of the root thus far secured is now doubled (72) and placed at the left of 5824 for a new divisor. The process then goes on as before, and it is found that if 8 becomes the next figure of the root, making the divisor 728, 728 will go into 5824 exactly 8 times. The square root of 135424 is therefore 368.

The square root of a decimal is extracted in the same way, marking off the periods as explained in a preceding paragraph:

```
        ______________
       √ 3'24.56'70'00 ( 18.015+
         1
        ___
 28 )    2 24
 ____
         2 24
        ______
    3601)    56 70
    _____
             36 01
            _______
   36 025)20 69 00
   ______
           18 01 25
          __________
            2 67 75
```

The square root is 18.015 to five significant figures. Notice that new periods of ciphers are added to the decimal as required.

Note also that the square of any number which is entirely a decimal will contain twice as many decimal places as the root.

It is true that in any arithmetical computation in which only five significant figures are required in the result, the use of no more than six significant figures in the elements of the computation is needed to secure this degree of accuracy in the result. That is, the use of one more significant figure in the elements of computation than the number of significant figures required in the result will be accurate enough.

Therefore, in extracting the square root of a "long" number, if only five significant figures are required in the root, use only six significant figures (the first six) of the number, replacing the remaining digits by ciphers. This will greatly simplify the computation and save much work. Thus, if the square root of 35,708,902 is required to five significant figures, extract the square root of 35,708,900; or, for the square root of 392,816,284.567, extract the square root of 392,816,000, and in each case the root will be correct to five significant figures.

To extract the square root of a fraction, the whole of which (that is, not just the numerator or denominator alone) is under the radical sign, extract the square root of the numerator and denominator separately:

$$\sqrt{\frac{4}{25}} = \frac{\sqrt{4}}{\sqrt{25}} = \frac{2}{5}$$

Exercises (correct answers at end of book):

B2) Extract the square roots of 196; 277,729; 795.24.

(B3) Extract the square roots of 49/81 and 225/256.

(B4) Extract the square root of 235,792.826 correct to five significant figures.

NOTE: There are arithmetical methods for extracting third, fourth, and fifth roots, but these are especially complicated. The average student will find little use for these—too little use, it is felt, to take space to explain them here. Besides, if the student does need these, he will find that the same effort spent on learning how to use tables of logarithms will not only enable him to find these roots approximately (enough for all practical purposes), but will help in many other computations as well.

III. INTEREST

The computation of interest on investments is an extension of the applications of percentage (see Percentage, Part I). It is, of course, a strictly commercial computation — but one which more intimately concerns the average man than most forms of bookkeeping and accounting.

Interest is the payment for the use of borrowed money. In olden times this payment was regarded as sinful, and was called *usury*. There is no longer any moral taint attached to the loaning of money at interest, providing the interest is reasonable and legal. *Usury* now applies only to exorbitant or illegal interest (in most states and countries a legal maximum rate of interest is established by law).

Interest is figured on a certain sum of money, called the *principal*, at a certain *rate* expressed in percent, for a certain length of *time*. Unless particularly specified otherwise, a rate of interest applies for one year—that is, interest is figured at a certain percent on the basis of one year's time. For example, if interest at the rate of 6% is paid on one dollar, it is paid at the rate of .06 of a dollar per year, or 6c per year on the dollar. If $100 were loaned for three years at 6%, the interest would be $18—6c on each dollar for each year. This is simple interest, and 6% is probably the most simple rate of interest—for it figures nicely for fractions of a year, particularly for periods of three and four months, which are common commercial loan periods.

In banks, the discount clerks (clerks who attend to the "discounting" of notes—that is, clerks who accept the promissory notes of good makers and, subtracting the interest and fees from the face of the note (discounting it), credit the balance to the account of the depositor) have what are known as Interest Tables, in which interest is figured at various rates on dollars in multiples of ten. But most of us figure interest so little that we must do it in our heads.

Simple interest ranges usually from 3% to 8%—on loans it is most commonly 6%, 7%, or 8%. Savings banks often pay peculiar rates, such as 4¾%—which is usually what is known as *compound* interest. Simple interest figures the amount paid on the principal, by multiplying the principal by the rate of interest expressed as a decimal. Thus, the interest on $52,900 at 7% would be:

$$\begin{array}{r} \$52{,}900 \\ \times \quad .07 \\ \hline \$3703.00 \end{array}$$

The interest for one year is thus $3,703; for three years it would be three times as much; for six months it would be one-half as much; for three months, one-quarter as much; etc. In simple interest, the interest is usually paid each year to the owner of the principal—in other words, the interest is not added to the principal, but the principal always remains the same.

In compound interest, the interest is added to the principal, and a new principal (equal always to the old principal plus accrued interest) is used as the basis of each successive computation. Thus, supposing a man left a principal of $52,900 at 7% compound interest (a very good rate indeed) for three years:

$$\begin{array}{r}
\$52,900 \\
\times \quad .07 \\
\hline
\$3703. \\
+52900. \\
\hline
\$56603. \\
\times \quad .07 \\
\hline
\$3962.21 \\
+56603. \\
\hline
\$60565.21 \\
\times \quad .07 \\
\hline
\$4239.5647 \\
+\$60565.21 \\
\hline
\$64804.77
\end{array}$$

His principal at the end of three years is therefore $64,804.77—a gain of $11,904.77; as he would have been paid only $11,109 at simple interest, the difference between simple and compound interest in this particular case is $795.77 net. Compound interest, as can be seen, figures interest on interest.

In general, investments in interest-bearing securities are at simple interest, for the value of the investment depends on the annual income derived from the interest payments. Savings deposits, on the other hand, are at compound interest, for if the depositor leaves his interest in the bank, he is, in effect, loaning that much more money to the bank, and should be paid interest on it. However, unless compound interest is specified, simple interest is always understood to be the method of computation.

A great many business concerns charge interest on overdue accounts. If they do business on a basis of 30 days credit, they may charge interest, at the legal rate, if a debtor allows his account to remain unpaid for, say, 60 days. Interest, let us suppose, is charged at the rate of 6% on an account of $57.28, which has run for 60 days and is therefore 30 days overdue:

```
     $57.28
    ×   .06
    -------
 12)3.4368
    ------
     .2864
```

Interest at 5% would be $3.44 (adding a cent since the fraction of a cent is more than ½c) for a year, so for 30 days (one month) it is one-twelfth of this, or 29c. By this addition of interest, the debtor now owes $57.57 instead of $57.28, and perhaps he allows this to run for another month without paying anything on it. The bookkeeper now adds another month's interest, but he must be very careful to figure the interest on the *original* amount—and not charge the customer interest on interest, for this is illegal. As it happens, on this particular account 6% of $57.57 would be only a matter of decimal parts of a cent as distinguished from 6% of $57.28—but the principle of the computation remains the same, for on a large account it would be a matter of dollars.

Exercises (correct answers at end of book):

(B5) Compute the simple interest on $567.35 at (a) 4%; (b) 4½%; (c) 6%; (d) 8%. The time, unless specified, is always understood to be one year.

(B6) Compute the interest on $2,800 at 7% for three months.

(B7) If on a principal of $10,000 a man is paid interest over a period of four years at the rates of 6%, 7%, 7½%, and 6¾% respectively, what is his average yearly income?

(B8) If $4,964 is left at 4¾% compound interest for 3 years and 9 months, what will the principal be at the end of that time?

(B9) How much would an initial principal of $5,000 earn at 5% compound interest for five years?

(B10) Mr. Jones has had hard luck, and hasn't paid his account with Blim & Blim for three months. Blim & Blim are strict adherents to a 30-day credit basis, and charge 8% interest on all overdue accounts. If Jones owed $782.15 when he made his last bulk purchase, having settled at that time all previous indebtedness, how much does he owe now, at the end of three months?

IV. GAIN AND LOSS

Another important function of percentage is to state the amount of gain or loss in a given transaction. If it is said that James Brown lost 15% on a real estate deal, what is meant by it? This: That Mr. Brown bought some land at a certain price, perhaps $15,000, and later sold it for less than he paid for it. We are told that he lost 15%, which means that he lost 15% of the purchase price, or 15% of $15,000. He therefore sold it for $15,000 less 15% (of $15,000) or:

$$\begin{array}{r} \$15{,}000 \\ \times\quad .15 \\ \hline 750.00 \\ 1500\ 0 \\ \hline \$2250.00 \end{array} \qquad\qquad \begin{array}{r} \$15{,}000 \\ -2{,}250 \\ \hline \$12{,}750 \end{array}$$

The selling price was therefore $12,750.

Or the problem may be stated in another way: Gregory Hilton is a man who makes a substantial income by buying property, "fixing it up," and selling at a profit. He purchases a house, two-family, for $12,000—paying $5,000 cash, and settling the balance with two notes, each for $3,500, the first for a period of three months, the second for six months, the current interest rate being 7%. The house needs many repairs, such as painting inside and out, new paper in some of the rooms, new lighting fixtures, some changes in plumbing, and so on—and the total cost of these reaches $1,895.56.

Hilton pays all of this within 30 days, except the plumber's bill, which is $395.00, and which he allows to run for four months, or until it is three months overdue. The plumber, being a business man, charges Hilton interest at 6%. The house was empty of tenants when Hilton bought it, and so remains while being repaired and for some time thereafter—three months in all. Hilton finally rents the lower tenement in June, and, since he is to supply heat to both tenements, he charges $50 a month rent during the summer months (May to October, inclusive), and $65 during the winter months. He does not rent the upper tenement, for which he charges the same rent, until December 1st. If through the ensuing winter months he burns a total of 19 tons of coal at $17.50 a ton, and his tenants remain with him until a year from the time of his purchase, and he then sells the house for $18,000, what is Hilton's percent of profit on his transaction?

This is a problem which the student must first analyze, and then solve step by step. The thing which must be figured first is, of course, the net cost to Hilton, including all of the items of expense. First of these is the purchase price of the house: $12,000. And to this we must add the interest on Hilton's two notes:

$$\begin{array}{r} \$3{,}500 \\ \times\ \ .07 \\ \hline 4)\ 245.00 \\ \hline \$61.25 \end{array} \qquad\qquad \begin{array}{r} 2)245.00 \\ \hline \$122.50 \end{array}$$

For three months, the first note costs Hilton

\$61.25, and for six months the second costs \$122.50—adding these to the purchase price:

$$\begin{array}{lr}
 & \$12,000 \\
 & 61.25 \\
 & +\ 122.50 \\
\hline
 & \$12,183.75 \\
\text{Repairs} & +\ 1,895.56 \\
\hline
 & \$14,079.31
\end{array}$$

We add also the cost of the repairs, as above. But Hilton has to let the plumber's bill run, and pays interest on that account:

$$\begin{array}{r}
\$395. \\
\times\quad .06 \\
\hline
4)\ \overline{23.70} \\
\hline
\$5.925
\end{array}$$

Since it was three months overdue, we add interest for three months at 6%, or \$5.93 net:

$$\begin{array}{r}
\$14,079.31 \\
+\quad 5.93 \\
\hline
\$14,085.24
\end{array}$$

Figuring back, we find that Hilton bought the house in March, assumably about the first of March. That means, if he kept it a year, that he sold it on the following March 1st. While on the items of expense, let us add the cost of his coal:

$$\begin{array}{r}
\$17.50 \\
\times\ \ 19 \\
\hline
157\ 50 \\
175\ 0 \\
\hline
\$332.50
\end{array}
\qquad\qquad
\begin{array}{r}
\$14,085.24 \\
+\quad 332.50 \\
\hline
\$14,417.74
\end{array}$$

Hilton now has some income from his tenants, which is to be subtracted from his cost. The first tenant comes in in June, and pays the summer rate of $50 from June to October, or for five months—$250. For November, December, January, and February, he pays $65—$260. The second tenant does not come in until December, so pays $65 for three months—$195. Adding these, and subtracting from the cost:

$$\begin{array}{r} \$250. \\ 260. \\ +195. \\ \hline \$705. \end{array} \qquad \begin{array}{r} \$14{,}417.74 \\ -\ 705.00 \\ \hline \$13{,}712.74 \end{array}$$

Since Hilton sells the house for $18,000, he makes:

$$\begin{array}{r} \$18{,}000. \\ -13{,}712.74 \\ \hline 4{,}287.26 \end{array}$$

His net cost is $13,712.74, so the percentage of gain must be figured on this as a base:

$$\begin{array}{r} 13{,}712.74\overline{)4{,}287.26} \\ .312 \end{array}$$

$$\begin{array}{r} 1371274\overline{)428726.000} \\ 411382\ 2 \\ \hline 7343\ 80 \\ 13712\ 74 \\ \hline 3631\ 060 \\ 2742\ 548 \\ \hline \end{array}$$

Or Gregory Hilton made about 31% on this transaction—not at all bad.

Rules: To find the percent of loss, divide the net loss by the net cost (or base, whatever it may be); to find the percent of gain, divide the net gain by the net cost (or base, whatever it may be). Obviously, the base may not always be dollars and cents. It may be desired to figure the gain or loss in the output of a factory, or in the number of gallons of water lifted by a pump, and so on. In these latter cases the base would become the original (before the loss or gain is recorded) total of automobiles, or gallons, or whatever the article may be. Put in another way, to find the percent of gain or loss, subtract the *initial* from the *final* (or vice versa) value, and divide the *difference* by the *initial* value.

Exercises (correct answers at end of book):

(B11) If a man buys a house for $5,200, pays cash, and loses 18% when he sells it, what is the selling price?

(B12) If a huckster fills his cart with fruit and vegetables at a cost of $111.56, and in two days sells everything but a few that spoiled for $135.28, what is his percent of profit?

(B13) A large department store puts in a stock of the Chinese game Mah Jongg, costing $3,000, with transportation charges amounting to $125.38; decorations for a Christmas booth to display the game, $12.97; a clerk to demonstrate and sell the game, four weeks' wages at $30 a week; delivery expense on customers' purchases approximately 9% of the total sale price; bookkeeping expense about 5%; and loss from breakage and wear about 3%—if the entire stock is marked to sell for $4,230, and is completely disposed of, what percent of net profit did this department show for this one venture?

(B14) A farmer strikes out into new proportions by buying a new farmhouse and several acres of land for $9,280. Not satisfied with the old barn, he tears this down and builds a new barn for just

21% of the total cost of his new house and land. If he pays an average interest rate on his total investment of 5½% for two years, and sells the farm for $10,000 to get clear in a hurry, what percent does he lose?

(B15) An engine of 180 horsepower loses 13% in friction. What is the actual horsepower available for use?

(B16) The driver of an automobile was able to make, at the most, only 33 miles an hour, until he added a patented attachment to his carburetor which lived up to its guarantee to the extent of increasing the speed of his car by 8%. If this attachment also increased the mileage per gallon of gasoline by 9%, and he was already getting 16 miles to the gallon, what was his maximum speed and his mileage per gallon with the new attachment?

(B17) If the publishing capacity of a concern printing little books is 120,000 copies every 8 hours and this must be increased 23% to keep up with the demand, what is the demand?

(B18) If a man in business estimates that out of every $100 invested he spends $75 for stock, $3 for selling expense, $4 for delivery expense, $3 for bookkeeping expense, $4 for expansion and improvement of his business property, $4 for insurance against fire and theft, $1.50 lost in bad accounts, $2.50 for taxes, and $3 for miscellaneous expenses, what percent of his cost (of goods) must he add to form the selling price of his goods, if he desires to make a net profit of 15%? (NET profit is distinguished from GROSS profit in that Net Profit is figured after all expenses have been deducted, and Gross Profit is merely the difference between the cost and the selling price, exclusive of incidental expenses.)

(B19) The simple interest on a principal of $20,000 netted, in four years, a gain of 28%. What was the rate of interest?

(B20) If the legal rate of interest in a certain state is 8% (maximum), and a dishonest loaning corporation makes a profit of $77 on a loan of $550, what percent of interest, in excess of the maximum legal rate, was charged?

V. DISCOUNTS

For various purposes in the business world there are *discounts* from the *list prices* of all kinds of wares. If I step into a shoe store to get me a pair of Sunday best, and the clerk whispers in my ear, "*To you* the price will be 10% off," he means, obviously, that for some fateful reason I am to get a discount of 10% from the usual selling price. Nearly every concern has its special list of discounts—if it sells paint, it probably gives 10% or so, to painters, that the ordinary customer does not get, and so on.

But there are certain established goods which have a universal selling price—a list price which may or may not be "cut" by the retailer. When the merchant buys these goods for his stock, he must, if he is to make any profit, buy them at a wholesale discount. This may be a simple discount, like 20%, 33⅓%, or 45% —20% might be shortened to 1/5 off, and 33⅓% to ⅓ off. Or it may be a more complicated discount, such as 16½%, or 23¾%. Or, again, it may be a chain discount, as 10-10-10% (10% and 10% and 10%—which DOES NOT equal 30%), or 60-40-20-10% (60%. 40%, 20%, 10%). A chain discount must always be taken in the order indicated. It is always possible to compute a single percent equal to the chain of discounts, by subtracting the first from 100, and taking the others from that. Thus, 60-40-20-10% would be:

$$\begin{array}{rrrr} 100 & 40 & 24 & 19.2 \\ -60 & -16 & -4.8 & -1.92 \\ \hline 40 & 24 & 19.2 & 17.28 \\ \times.40 & \times.2 & \times.1 & \\ \hline 16.00 & 4.8 & 1.92 & \end{array}$$

Which is to say that 60-40-20-10% if taken in succession, will be equivalent to taking 17.28% of the base at the start (or is equivalent to a discount of 82.72%). For to take a discount of 60% it is first necessary to multiply by .6, and then subtract the product from the base. If the base is multiplied by the difference between the discount and 100, or .4 (40%), the product is equivalent to the base with the discount subtracted. Thus:

$$\begin{array}{rrr} \$23.40 & \$23.40 & \$23.40 \\ \times\quad .6 & -14.04 & \times\quad .4 \\ \hline \$14.040 & \$9.36 & \$9.360 \end{array}$$

So, above, if 17.28% is taken of the base (the amount the discount is to be figured on), it will save subtracting 82.72% of the base from itself. Similarly, in computing the equivalent of 60-40-20-10%; if the multipliers had been .6, .8, and .9, respectively, all the subtraction would have been eliminated.

To take 60-40-20-10% from a list price of $44.65, either way will do—the long way of using each percent of the chain discount in turn, or the short way of using 17.28%:

$44.65
× .4

$17.860
× .6

$10.716
× .8

$8.5728
× .9

$7.71552

$44.65
× .1727

35720
8930
3 1255
4 465

$7.715520

(Wherever possible in decimal computations, final ciphers (at the end of the decimal) are dropped.)

In business parlance, a chain discount of 10-10% is often called "ten and ten" or "two tens," and 10-10-10% is called, similarly, "three tens." Such a simple chain discount is very easy to figure. Some chain discounts, such as that on cotter pins, are very peculiar affairs —although, likely enough, the manufacturers have a reason for it. Discounts in general, from fixed list prices, are born not only of a desire to keep the merchant's profit a dark secret from the retail purchaser—but of an honest effort to cut down expense. Catalogues of merchandise cost money, and if the wholesale prices were printed in each catalogue, entirely new electroplates would have to be made with every price change. But a catalogue with fixed list prices is a more or less permanent affair, and an attached "discount card" is not only a very simple means of providing an easily changed wholesale quotation, but permits offering different discounts to different dealers, or to different localities.

The average shopper meets with discounts in bargain sales. "CLEARANCE SALE: All Goods at 33⅓% Off," says the glaring sign. Or perhaps a counter full of heaped knickknacks is labeled: "10% off the marked prices on all these goods."

Exercises (correct answers at end of book):

(B21) If a second-hand automobile is offered at a discount of 35% from the purchase price of $2,400, plus a delivery charge of $100, what is the net cost to the purchaser?

(B22) If the list price of a book is $2.50, and the wholesale discount is 33⅓%, how much under the list price does a bargain bookseller offer the book if he makes a profit of 25%?

(B23) What single percent of discount is equivalent to a chain discount of 35-15-5%?

(B24) Is a change in the discount from 10-10-10% to a simple 30% on a certain class of goods an increase or decrease in the discount? How much is the difference?

(B25) A purchasing agent buys $456.75 (purchase price) worth of goods at a chain discount of 25-15-10%. What was the list price?

VI. WEIGHTS AND MEASURES

Weights and Measures are properly *denominate numbers* or *concrete numbers*, as opposed to abstract numbers. They deal with numbers of *things*—inches, quarts, seconds—what not? A simple denominate number deals with a single thing: 15 seconds. A compound denominate number deals with more than one thing more or less related: 3 minutes, 4 seconds.

The systems of weights and measures now in use in English-speaking countires (chiefly) are unwieldy and clumsy. They are unscientific, built by the slow and uncertain vagaries of years. Although familiar because of everyday use, they lack the advantage of easy interchangeability, and involve all kinds of queer multipliers and strange equivalents. These difficulties are overcome by the Metric System, which will be explained a little later—a system in accordance with the decimal monetary system of the United States (but England has an unscientific monetary system in accordance with these archaic weights and measures).

Measures may be divided into various classes: those of length, area, and volume (*extension*); those of weight; those of capacity (dry and liquid); those of time; those of angles (arcs); and those of money or coins (*value*). They involve not only the current measures of everyday transactions, but a great many old and nearly obsolete units. Tables of these, with conversion equivalents, are given in *A Hand-*

book of Useful Tables (Little Blue Book No. 835).

But for the convenience of the student some of the more common tables of measures are given here:

Length

12 inches = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards = 1 rod
320 rods = 1 mile (m.)

Area

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
128 cubic feet = 1 cord

Weight (Avoirdupois)

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)
2240 pounds = 1 long ton

Weight (Troy)

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce
12 ounces = 1 pound

Capacity (Dry)

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Capacity (Liquid)

4 gills (gi.) = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day
7 days = 1 week (wk.)
52 weeks = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
100 years = 1 century

Angles

60 seconds = 1 minute
60 minutes = 1 degree
90 degrees = 1 right angle
360 degrees = 1 circumference (circle)

Money (U. S.)

10 mills (m.) = 1 cent (c)
10 cents = 1 dime
10 dimes = 1 dollar ($)
10 dollars = 1 eagle

Money (British)

12 pence (d.) = 1 shilling (sh.)
20 shillings = 1 pound (£)

Miscellaneous

12 units = 1 dozen (doz.)
12 dozen = 1 gross
12 gross = 1 great gross
2 units = 1 pair
20 units = 1 score

Denominate numbers may be reduced from higher to lower denominations, or from lower to higher. Thus 3 yards may be reduced to 9 feet (3 yards = 9 feet), or 9 feet may be reduced to 3 yards (9 feet = 3 yards). Depending upon circumstances, it may be desirable to keep a measurement entirely in inches, or to keep it in yards, feet, and inches. And so, too, with the other measures.

The student should know at least all of the tables given here by heart. To help him to familiarize himself with them, a few exercises are given (correct answers at end of book):

(B26) Reduce to lower denominations 20 rods, 4 yards, 2 feet, 8 inches; 5 gallons, 3 quarts, 1 pint; 4 pounds; 3 days, 10 hours.

(B27) Reduce to higher denominations 760 seconds; 2,534 cents; 148 dozen; 46 pints (dry); 57 pints (liquid); 542 grains (troy); 246 ounces (avoirdupois); 3,488 cubic inches; 764 square inches; 2,378 inches.

(B28) If potatoes are 66c a peck, what would be the cost of 4 bushels?

(B29) If there are 96 pieces of candy in two pounds, how many pieces are there in 1 ounce?

(B30) If a man burns a pint of gasoline to drive his automobile 2 miles, and gasoline is 23c a gallon, how much does his trip cost per mile?

(B31) Add 4 yards, 2 feet, 9 inches and 3 rods, 3 yards, 1 foot, 7 inches.

(B32) Subtract 3 pints from 2 gallons.

(B33) If to sprinkle a road costs 25c a foot per year, how much does it cost to sprinkle a quarter of a mile?

(B34) Multiply 5 bushels, 2 pecks, 1 quart, by 3.

(B35) Divide 1 square rod, 3 square yards, 4 square feet, by 2.

THE METRIC SYSTEM.—The metric system is a system of weights and measures based on decimal equivalents. It is used today in several countries (particularly in France) in all measurements, and in all countries for scientific measurements. The student of even elementary arithmetic cannot afford to be ignorant of it.

The metric system presents difficulties only in the names of the units—these are a little strange. If it were not for the familiarity of the older units in America and England, the metric system would readily be acknowledged as far the easier and simpler. That it is both easier and simpler everyone who *knows* it will grant.

The standard of length is the *meter*, supposed to be the distance of one ten-millionth of the earth's surface from the equator to the pole—it equals, in the so-called English system, about 39 inches, or slightly more than a yard. The names of the other units of length are merely variations of the word meter, made by prefixing certain Greek and Latin forms which indicate the unit's relation to the meter—*deca* for 10 times, *hecto* for 100 times, *kilo* for 1000 times; *deci* for one-tenth; *centi* for one-hundredth; *milli* for one-thousandth. These prefixes are used throughout the metric system, for weights, capacity, and so on.

Length

10 millimeters = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 decameter (Dm.)
10 decameters = 1 hectometer (Hm.)
10 hectometers = 1 kilometer (Km.)

Area

100 square millimeters = 1 square centimeter (cm.^{2})
100 square centimeters = 1 square decimeter (dm.^{2})
100 square decimeters = 1 square meter (centare)
100 square meters = 1 square decameter (are)
100 square decameters = 1 square hectometer (hectare)
100 square hectometers = 1 square kilometer

Volume

1000 cubic millimeters = 1 cubic centimeter (cm.^{3})
1000 cubic centimeters = 1 cubic decimeter (dm.^{3})
1000 cubic decimeters = 1 cubic meter
1000 cubic meters = 1 cubic decameter
1000 cubic decameters = 1 cubic hectometer
1000 cubic hectometers = 1 cubic kilometer

Weight

10 milligrams = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram
10 grams = 1 decagram
10 decagrams = 1 hectogram
10 hectograms = 1 kilogram
100 kilograms = 1 quintal

Capacity

10 milliliters = 1 dentiliter
10 centiliters = 1 deciliter
10 deciliters = 1 liter
10 liters = 1 decaliter
10 decaliters = 1 hectoliter
10 hectoliters = 1 kiloliter

Conversion Equivalents

1 meter = 39.37 inches
1 kilometer = .62 mile
1 square meter = 10.76 square feet
1 hectare = 2.47 acres
1 cubic centimeter = .06 cubic inch
1 liter = 1.05 quarts (liquid)
1 liter = .9 quart (dry)
1 kilogram = 2.2 pounds
1 inch = 2.54 centimeters
1 foot = .3 meter
1 mile = 1.6 kilometers
1 square inch = 6.45 square centimeters
1 cubic inch = 16.39 cubic centimeters
1 gallon = 3.78 liters

Exercises (correct answers at end of book):

(B36) If the dimensions of a room are 14 feet by 10 feet by 8 feet, what are the dimensions in meters?

(B37) If a block of granite weighs 50 kilograms, what is its weight in pounds?

(B38) A French farm contains 48 hectares. How large a farm would this be in American acres?

(B39) A drive along the Western front, extending for 15¾ miles, would extend for how many kilometers?

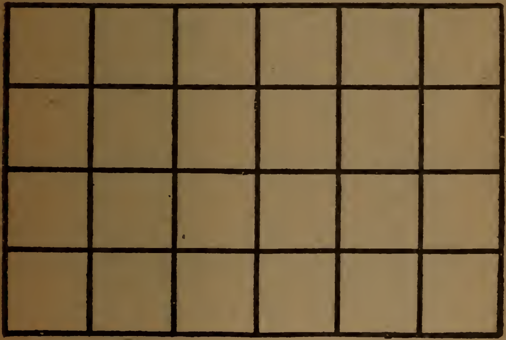
(B40) From the above equivalents calculate the English equivalents of the centimeter, the gram, the cubic meter; the metric equivalents of the yard, the square foot, the bushel, and the pound (the last two must be calculated by using the English equivalents of the liter and kilogram).

VII. AREA

The world in which we live is in *three dimensions*—length, breadth, height—"up, down, and across." Every dimension is one of these, or some combination of these. A stick, a piece of string, a road—each has length as its most important dimension. In these days of whizzing automobiles, the width of a road is coming to be of great importance also. But a road, strictly speaking, has no height, no thickness. A road is therefore in two dimensions only. But this is because we think of a road as some surface without thickness—actually it does have thickness. There is no such thing, actually, as a piece of string, or anything, with only length for a dimension. The tiniest thread of a spider's web has length — and breadth or thickness too (for it is circular). If it didn't, it wouldn't exist. The thinnest piece of paper has two dimensions—and also thickness, making three. But, for practical purposes, a thin piece of paper has no thickness; the thickness, that is, is immaterial—it doesn't count. Similarly, in measuring a piece of string, its length, one dimension, is usually all that interests us. So, although everything we know has three dimensions, we regard many things as measured in only one or only two dimensions, because this is all that is needed for our purposes.

In geometry, however, the student will find that figures are theoretically drawn in exactly two dimensions—without thickness. A line in geometry has only length—neither width nor thickness. Of course, the line must be drawn with something, pen or pencil or chalk, so it does have an actual width and an actual thickness of ink or lead or chalk particles, but theoretically it has only length. For geometrical purposes it is necessary to imagine lines which have only one dimension each. Too, plane figures in geometry have only two dimensions—length and breadth. But in solid geometry the figures have three dimensions. Not content with three dimensions, the mathematicians have pursued their theoretical science to the point of imagining a *fourth dimension*—if there can be three, why not four? They say that perhaps there is a world which is built in four dimensions, as ours is built in three. In four dimensions it is contended that a rubber ball can be turned inside out without cutting it in any way! But the fourth dimension is a bit beyond the practical limits of this book, so we'll confine ourselves to three.

Area is the amount of surface contained within two dimensions. The simplest surface to measure for area is one bounded by four straight lines, meeting each other at right angles (like the corners of a square)—called a rectangular surface. The top of a rectangular table, the floor or ceiling of a square or rectangular room—or, better, the top, bottom, or one side of a box each surface of which is bounded by four straight lines—may serve as an example. Taking a surface of the box, each edge of this surface has length: it may be measured, according to its size, in inches, feet, or yards. Supposing that inches will fit our box best, we can mark off, along one edge, the number of inches it contains. Supposing, further, that one edge contains exactly four, and the other exactly six inches, we can divide the entire surface into little squares one inch wide and one inch high:



The dimensions of this surface (ignoring the thickness of the box material, for we are considering only the *surface*, which is theoretically in only two dimensions) are 4 inches by 6 inches. Counting the little squares, we find that there are exactly 24 of them—and each square, since it measures one inch on each side, is called a *square inch.* If each square measured a foot on a side, it would be a square foot; if each measured a yard on a side, it would be a square yard; and so on. This surface therefore contains 24 square inches—or its area is said to be 24 square inches:

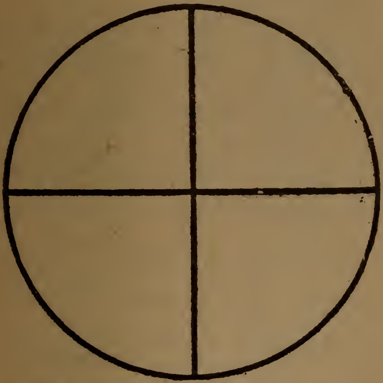
$$\text{Area} = 24 \text{ sq. in.}$$

But when we desire the area of such a surface, we cannot always stop to divide it into squares and count the squares. There is a shorter way than this. Since one edge of this surface measures 4 inches, there are 4 square inches along this edge. Along the other edge, in similar wise, there are 6 square inches. Obviously, then, to complete the surface, there must be 4 rows of 6 square inches each (or 6 rows of 4 square inches each, whichever way one cares to look at it), or there are 4 × 6 square inches = 24 square inches. We can therefore formulate a rule (for we took as an example *any* rectangular surface, so this must always hold true): *To find the area of any right-angled surface, multiply the length by the breadth, both length and breadth being expressed in like units of length, the area being expressed in the square of that unit.*

The length and breadth must be expressed in *like* units of length, for it would not do to multiply yards by feet. Either both would have to be in yards, or both in feet. For odd measurements, simply use the unit that is most convenient. Thus, to find the area of a floor measuring 3 yards, 2 feet, 9 inches, by 4 yards, 1 foot, 8 inches—change to feet. The dimensions then are 11¾ feet by 13⅔ feet, and the area is the product of these:

$$\text{Area} = 11\tfrac{3}{4} \text{ ft.} \times 13\tfrac{2}{3} \text{ ft.} = 153\tfrac{1}{4} \text{ sq. ft.}$$

The preceding rule specifies that the surface must be right-angled. That is, each corner (and there must always be four corners) must contain 90°, or one-quarter of a circle. When two lines meet at right angles to each other, they are said to be *perpendicular*—if a circle is quartered, as in the accompanying figure, the two lines which pass through its center are *diameters*, and these diameters are perpendicular to each other. Where they meet there are then four right angles:



If a surface is not entirely right-angled, this particular rule does not apply. A right-angled surface is either a *square* (all sides equal) or a *rectangle* (see Plate I, Page 45). The surface may have only three sides and three angles, instead of four—it is then a *triangle*. Or it may have four sides, and be slightly askew, as in a *parallelogram* that is not right-angled (squares and rectangles are right-angled parallelograms, for opposite sides are *parallel*). Or one pair of opposite sides may be parallel, and the other pair not—when it is a *trapezium*. Or the surface may be entirely bounded by a regular circular line, and be a *circle*. Or the curved line may be somewhat more long than broad, and be an *ellipse*. For the areas of these various surfaces, other rules apply:

PARALLELOGRAM: *Area* = *base* (bottom line) × *altitude* (height, measured in a line perpendicular to the base line).

TRIANGLE: *Area = one-half the base × altitude.*

TRAPEZIUM: *Area = one-half the product of the sum of the two parallel sides × the distance between them.*

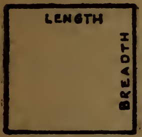
CIRCLE: *Area = square of the diameter* × .7854.

ELLIPSE: *Area* = .7854 × *the long axis* × *the short axis.*

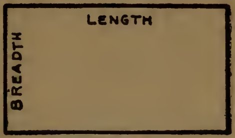
If it is desired to measure the area of the surface of a *sphere* (any regular ball or globe, perfectly round, without irregular bumps or dents), the *diameter* (length of a line drawn straight through the center) must be known. The area of the surface then is:

SURFACE OF A SPHERE: *Area* = 3.1416 × *diameter squared.*

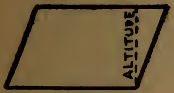
PLATE I



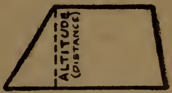
SQUARE



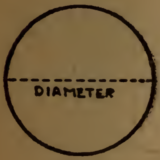
RECTANGLE



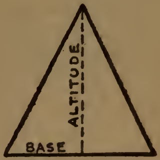
PARALLELOGRAM



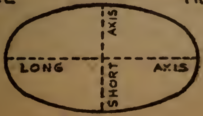
TRAPEZIUM



CIRCLE



TRIANGLE



ELLIPSE

Exercises (correct answers at end of book):

(B41) What is the area of a square field measuring 48 feet on a side? Express this as a decimal fraction of an acre (see table under Weights and Measures for equivalent of an acre).

(B42) If a rectangular kitchen floor has for its dimensions 9 feet, 7 inches, by 5 feet, 11 inches, how many square yards of linoleum would be needed to cover it, assuming that the linoleum could be obtained in the exact quantity desired to fit the floor.

(B43) If a small homestead of 14 acres measures 360 feet along one side, and is approximately rectangular in shape, how long is the other side?

(B44) How many square inches of cloth would be needed to cover all the sides of a square box measuring 7 inches on each edge?

(B45) If a man is hired to cut a triangular plot of grass for the city, and is paid $20 for cutting it twice a week, how much is he paid per square yard, if the plot measures 35 feet along the base and the length of a line drawn from the tip of the triangle perpendicular to the base (or the length of the altitude) is 45 feet?

(B46) If the area of a triangle is 176 square inches, and one-half the base is 11 inches, what is the altitude?

(B47) If a circus clown jumps through four paper-covered hoops a day, each hoop exactly circular and measuring 4 feet in diameter, how many square feet of paper does he destroy in a week, assuming that he jumps 6 days out of the 7?

(B48) A city park builds a pond exactly in the shape of an ellipse. If the long axis of this pond measures 49½ feet, and the short axis 32 feet, how many square feet of ice will the pond offer for skating when it freezes over?

(B49) A farmer is informed that the floor area of a circular silo is about 50¼ square feet, but he wants to know what the width, or diameter, of this silo is, in order to know whether it will fit within a certain space by his barn. What is the diameter?

(B50) To aid in driving in and out with his three cars, a millionaire has built his garage in the shape of a trapezium, the front measuring 40 feet, and the rear 30½ feet, the perpendicular distance from front to back measuring 35 feet. How many square feet of cement surface is there on the floor of this garage?

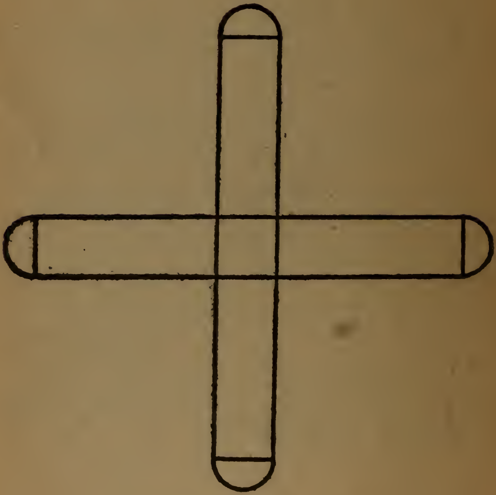
(B51) How many square inches of rubber surface are there on a child's large rubber ball which has a diameter of 3½ inches?

(B52) The earth's diameter is roughly figured at 8,000 miles. Assuming that the earth's surface is approximately smooth, how many square miles of surface does the earth have?

(B53) The dome of a state capitol is exactly half of a sphere, with a diameter of 25 feet. Assuming that the amount of gold-leaf needed to cover the round surface of this dome is exactly equal to its surface area, how many square inches of gold-leaf will be required?

(B54) A man with a hobby for garden plots of strange shapes, makes one that is composed of a rectangle measuring 11 feet by 4½ feet, with a triangle built on each short side of the rectangle for a base, each triangle measuring 3 feet from tip to base. What is the area of this plot?

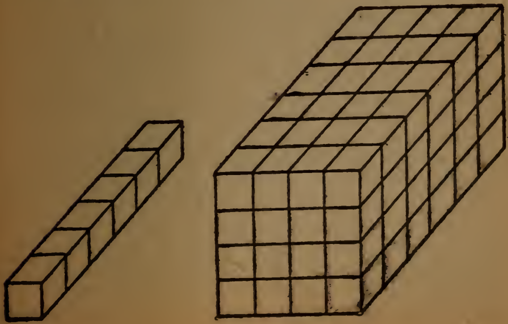
(B55) Another plot consists of a Greek cross, each bar of which is a rectangle 39 feet long by 5 feet wide; and on each end of the cross there is added a semicircle. What is the total area of this plot? (Hint: Study the accompanying diagram on page 48 carefully; note that where the bars of the cross pass over each other there is a square 5 feet on a side, the remaining bars forming four rectangles; the end-lines of the bars form the diameters of the semicircles.)



(Diagram for Problem B55)

VIII. VOLUME

Just as area is the measure of the surface of something, so *volume* is the measure of solid contents or space contained *inside* of an object of three dimensions. Taking for an example the rectangle measuring 4 inches on one side and 6 inches on the other (under Area), let us suppose that four surfaces like this form four sides of a rectangular solid, the two ends measuring 4 inches by 4 inches (for if the four surfaces form four sides of the "box," each group of four ends of these four surfaces must form the edges of the ends of the box). Put in another way, this box is made up of 16 smaller boxes, each measuring 6 inches long, with ends of 1 square inch each. For the end of our big



box can be divided, as shown, into 16 small squares, each 1 square inch. Since each edge of the box is 6 inches long, lengthwise, there are 6 square inches along each edge. Supposing that each of these smaller boxes is made up of *cubes*, measuring 1 inch on each edge, these cubes represent each a *cubic inch*. Each of these 16 smaller boxes then contains 6 cubic inches. The entire big box must then contain 6 × 16 cubic inches = 96 cubic inches. But this is the product of the three dimensions of the box: 4 × 4 × 6 inches = 96 cubic inches. Just as in the preceding a cubic inch measured an inch on a side, so a cube measuring a foot on a side would be a cubic foot, and one measuring a yard on each side would be a cubic yard, and so on. The rule for finding the volume of a rectangular solid of six sides is therefore: *Volume equals the product of the three dimensions, expressed in like units, the results being in cubic units.* Technically, this rule applies only to *rectangular prisms.*

The volumes of solids that do not come under this rule may usually be computed in fairly simple ways, unless the solid is very irregular. Understand that by the term *solid* is meant any figure or space which must be defined in three dimensions—an empty room, without irregularities, would have six sides, right angles at all the corners, and would be a solid in this sense. Probably the next most common solid is a *sphere*, of which the surface area has already been discussed. A *cylinder* is a solid with two flat circular bases and a round surface connecting them—in short, a section of a rod or pole cut off at each end at right angles to the edge.

A *prism* that is not rectangular is perhaps triangular, or octagonal, depending on the number of sides of the base. Any solid is a prism which has for two like and equal bases some *regular polygon* (a geometrical figure with equal sides and equal angles—more than four sides always), and parallelograms for *faces* or side-surfaces. A *pyramid* is a solid with any regular polygon for a base, and triangular faces culminating in a point. A *cone* is very similar to a pyramid, except that it has a circle for a base and a continuous circular surface tapering to a point above the center of the base. Formulas for the volumes of these various solids, which are graphically represented on Plate II, Page 53, follow:

PRISM: *Volume = area of the base × altitude* (in a rectangular prism, the height and altitude are identical; otherwise, the altitude is a line drawn perpendicular to the lower base and up to the upper base, or top). A prism may be "slanting," i. e., have non-rectangular parallelograms for faces.

SPHERE: *Volume* = 4.189 × *cube of the radius* (radius = one-half the diameter).

CYLINDER: *Volume* = 3.1416 × *square of the radius of the base × altitude* (alt. = height).

CONE: *Volume* = 1.047 × *square of the radius of the base × altitude* (Alt. = perpendicular distance from apex to center of base).

PYRAMID: *Volume = one-third area of the* *base* × *altitude* (Alt. = perpendicular distance from apex to center of base).

Exercises (correct answers at end of book):

(B56) If the dimensions of a rectangular room are 40 feet, 6 inches, by 28 feet, 8 inches, by 11 feet, 4 inches, how many cubic feet of air does the room contain?

(B57) How many cubic feet of earth must be dug out to make an excavation 10 feet deep by 28 feet 3 inches wide, by 16 feet 9 inches long?

(B58) Knowing that the diameter of the earth is 8,000 miles, what is its approximate bulk in cubic miles?

(B59) What is the capacity in cubic feet of a farmer's cylindrical silo, which is 40 feet high and measures 2 yards across the base?

(B60) If an ice-cream cone measures 5 inches long, and the diameter of the opening is 1½ inches, how many cubic inches of ice cream would it hold if crammed full?

(B61) The Great Pyramid of Egypt has a square base measuring 756 feet on a side, and is 481 feet high. Assuming that the pyramid is of solid stone, and ignoring the fact that it has been cut off at the top so that there is a flat surface there 33 feet square—how many cubic feet of stone does this eternal monument contain?

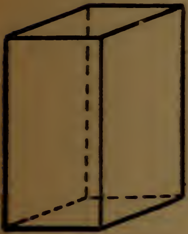
(B62) To put out a small fire in a small town, suppose 450 buckets of water were used. If each bucket is equivalent to a cylinder 18 inches high and with a diameter of 11 inches, and assuming that each bucket was ¾ full, how many cubic feet of water did it take to drown the fire?

(B63) If a man cuts 75 blocks of ice a day, each measuring 12 inches by 15 inches by 18 inches, and his ice house holds 1,200 cubic feet of ice, how many days will it take him to fill his ice house?

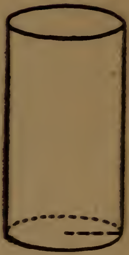
(B64) If a cubic foot of water weighs 62 pounds and a large tank holds 6,975 tons of water, what is the contents of this tank in cubic feet?

(B65) If the tank in the preceding problem has a base 75 feet by 60 feet, what is its height?

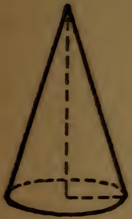
PLATE II



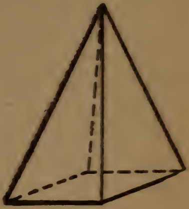
PRISM



CYLINDER



CONE



PYRAMID

IX. PRACTICAL PROPORTION

Proportion has already been treated in Part I, under the general heading "Ratio and Proportion." But there is much to proportion and the good old *rule of three*, much that the student cannot well ignore. The practical applications are many and various.

Sometimes one comes across what are known as *inverse* ratios, and these involve *inverse* proportions. An inverse proportion is simply one that is inverted from the normal order. Thus, 10 is to 5 *inversely* as 4 is to 8 would have to be written: 10:5::8:4—that is, the second ratio would have to be inverted to match the first. When it is known or stated that one ratio is in inverse proportion to another, inverse ratios cause no trouble, for it is a simple matter to invert the second ratio to match. However, it is only by studying the conditions of some problems that it can be ascertained that one ratio *is* in inverse proportion to another—and there's where the trouble comes.

There is a little trick to this matter of inverse proportions which may help. As can be seen from all proportions, if the first term is larger than the second, then the third term must be larger than the fourth—or vice versa. So the student may first state his problem as a direct proportion. If his first term is larger than his second, then he may at once assume that, according to the proportion as he has stated it, his third term must be larger than his fourth. One of these terms is the unknown he is trying to find—since it is always wise, where possible, to make this the last term, we'll assume that his fourth term is the missing one. According to his proportion, this must be smaller than his third. A careful examination of the conditions of his problem may now tell him whether this is likely to be so. If it is, then his proportion is probably correct; if it isn't then his ratios are in inverse proportion, and one must be inverted to match the other.

Proportion is sometimes simplified by applying the principle of *cause and effect*. Anything producing a change in the condition of something else or accomplishing something as a consequence of itself, may be called a cause. The change or the thing accomplished will then be the effect. The principle then is: *Like causes produce like effects*. Expanding this, two causes of similar kind produce two effects of similar kind; or, more arithmetically, the ratio of the causes is equal to the ratio of the effects—the first cause is to the second cause as the first effect is to the second effect.

Example: If 4 men can carry 15 bushels of potatoes, evenly distributed, how many bushels can 7 men carry? Here the causes are obviously the men: 4 men and 7 men, first and second causes, accomplish something—the carrying of the bushels. The number of bushels carried by 4 men is the first effect, and the number carried by 7 men, the unknown quantity, is the second effect. Sometimes the unknown is best represented by the letter x, so, putting our ratios into a proportion:

$$4 : 7 :: 15 : x$$

Proceeding according to the rule of three, we find that x is equal to the product of 7 × 15 divided by 4, or 26¼ bushels.

All this has been simple proportion. In *compound proportion* two or more of the terms consist of more than one number each. The principle of cause and effect is used in a similar way.

Example: If 38 men earn \$976 in 13 days, how much will 34 men earn in 22 days? Here the causes are 38 men and 34 men, certainly, for they earn something, \$976, as an effect. But the time they take to earn it is another cause, so 13 days and 22 days are causes also, with \$976 as an effect. The effects are single for the double causes, being \$976 and an unknown number of dollars, represented by x. Arranging this in a compound proportion, with 38 men and 13 days as the first cause (compound), and 34 men and 22 days as the second cause (compound), we have \$976 as the first effect (simple), and x as the second effect (simple):

1st cause	*2d cause*		*1st effect*	*2d effect*
38	34	=	976	x
13	22			

The vertical lines between the first two and last two terms are equivalent to the customary colons and have the same meaning. They are more convenient for use in compound proportion. Now, proceeding according to the rule of three, all the figures inside the two vertical lines (34, 22, 976), if multiplied together, must equal the product of all the figures outside the vertical lines (38, 13, x), or:

$$\frac{34 \times 22 \times 976}{13 \times 38} = x$$

Canceling out, we have:

$$\frac{34 \times 11 \times 976}{13 \times 19} = \frac{365{,}024}{247} = \text{about } \$1{,}073 = x$$

Or, 34 men will earn $1,073 in 22 days.

If there happen to be more than two figures in any terms of a compound proportion, simply proceed on the same principle. Write all the figures of the first cause in a vertical column in the position of the first term, all the figures of the second cause in a vertical column in the position of the second term, and so on. Then all the figures within the vertical lines (the means), if multiplied together, must equal the product of all the figures without the vertical lines (the extremes).

Exercises (correct answers at end of book):

(B66) If 52 men can earn $1,897 in 27 days, how much can 43 men earn in 39 days?

(B67) If 38 men can dig a ditch 680 feet long, 4 feet wide, and 3 feet deep in a certain number of days, how long a ditch 5 feet deep and 3 feet wide can 20 men dig in the same number of days?

(B68) If a block of granite 10 feet long, 6 feet wide, and 4 feet thick weighs 40,800 pounds, how much does a block of granite 15 feet long, 9 feet wide, and 5 feet thick weigh?

(B69) If 10 compositors in 28 days of 8 hours each are able to set up 20 sheets of 16 pages each, 35 lines to the page, in how many days 7 hours long can 20 compositors set up, in the same size and style of type, 48 sheets of 12 pages each, 40 lines to the page?

(B70) If a man travels 490 miles in 15 days of 8 hours each, how many hours a day must he travel to go 580 miles in 20 days?

ANSWERS TO EXERCISES

(B1) 1024; 1000; 1; 2.25; 463.1104; 16/81
(B2) 14; 527; 28.2
(B3) 7/9; 15/16
(B4) 485.59—
(B5) (a) $22.69
(b) $25.53
(c) $34.04
(d) $45.39
(B6) $49.00
(B7) $681.25
(B8) $5,908.75
(B9) $1,381.41
(B10) $792.58
(B11) $4,264
(B12) About 21%
(B13) About 6% — rather poor
(B14) Almost 20% (figures .197+)
(B15) 156.6 horsepower
(B16) Maximum Speed 35.64 miles per hour; mileage per gallon 17.44 miles
(B17) 147,600 copies every 8 hours
(B18) 53 1/3%
(B19) 7%
(B20) 6% excess (14% in all)
(B21) $1,660
(B22) 41c
(B23) About 48% (exactly .4751—)
(B24) Increase of 2.9%
(B25) $796.08
(B26) 4136 inches; 47 pints; 64 ounces; 82 hours or 4920 minutes
(B27) 12 minutes, 40 seconds; $25.34; 1 gross, 4 dozen; 2 pecks, 7 quarts; 7 gallons, 1 pint; 1 ounce, 2 pennyweights, 14 grains; 15 pounds, 6 ounces; 2 cubic feet, 32 cubic inches; 5 square feet, 44 square inches; 66 yards, 2 inches
(B28) $10.56
(B29) 3 pieces
(B30) About 1½c (exactly $.014375)
(B31) 4 rods, 2 yards, 2 feet, 10 inches
(B32) 1 gallon, 2 quarts. 1 pint
(B33) $330
(B34) 16 bushels, 2 pecks, 3 quarts
(B35) 16 square yards, 7 square feet, 90 square inches
(B36) 4.2 meters by 3 meters by 2.4 meters
(B37) 110 pounds
(B38) 118.56 acres
(B39) 25.2 kilometers (1 m.=1.6 Km.)
(B40) 1 centimeter = .30 inch
1 gram = .035 ounce
1 cubic meter = 416.65 cubic feet
1 yard = .9 meter
1 square foot = .093 square meter
1 bushel = 35.2 liters
1 pound = .454 kilogram

(B41) 2304 square feet = .053— acre
(B42) 6 square yards, 2 square feet, 101 square inches (or at least 7 square yards)
(B43) 121 feet
(B44) 294 square inches
(B45) About 11c per square yard
(B46) 16 inches
(B47) 301.59 square feet of paper
(B48) About 1,244 square feet
(B49) A trifle less than 8 feet
(B50) 1233.75 square feet
(B51) 38.48 square inches
(B52) 201,062,400 square miles
(B53) 141,372 square inches
(B54) 63 square feet
(B55) About 404¼ square feet (exactly 404.27)
(B56) 13158 cubic feet
(B57) 4731 7/8 cubic feet
(B58) 268,096,000,000 cubic miles
(B59) About 1131 cubic feet (exactly 1130.976)
(B60) About 3 cubic inches (exactly 2.945—)
(B61) 91,636,272 cubic feet
(B62) 334.1+ cubic feet
(B63) plus days
(B64) 225,000 cubic feet
(B65) 50 feet
(B66) About $2265.86
(B67) 286.3+ feet

The proportion should look like this at the beginning:

			680	x
38	20	=	4	8
			3	5

(B68) 114,750 pounds
(B69) **Almost 83 days**

The proportion should look like this at the beginning:

8	7		35	40
10	20		20	48
28	x	=	16	12

(B70) About 7 hours and 6 minutes per day

INDEX.

(Arabic numerals refer to pages; Roman numerals signify Part I, Little Blue Book No. 856, or Part II, the present volume.)

abstract numbers, I-11.
addition, I-11.
angles, measure of, II-34.
area, II-33; II-39.
averages, I-60.
cancelation, I-34; I-49.
capacity (dry), II-33; (liquid), II-33.
cause (proportion), II-55.
circle, II-44.
concrete numbers, I-11; II-32.
cone, II-51.
cube (power), II-7.
cylinder, II-51.
decimals, I-51; addition of, I-14; division of, I-32; fractions, I-37; multiplication of, I-22; notation of, I-9; powers of, II-8; square root of, II-14; subtraction of, I-24.
denominate numbers, II-32.
denominator, I-37; least common, I-42.
difference, I-23.
discounts, II-28.
dividend, I-26.
division, I-26; long, I-27.
divisor, I-26.
effect (proportion), II-55.
ellipse, II-44.
evolution, II-10.
exponent, II-8.
extremes, I-62.
factoring, I-34.
factors, prime, I-34.
fractions, I-26; I-37; addition of, I-41; division of, I-47; multiplication of, I-46; powers of, II-8; proper and improper, I-38; reduction of, I-39; square root of, II-15; subtraction of, I-44.
gain, II-21.
hundreds, I-6.
integer, I-8.
interest, II-17; simple, II-18; compound, II-18.
inverse proportions, II-54.
involution, II-7.
length, measures of, II-38.
loss, II-21.
means, I-62.
measures, II-32.
metric system, II-35.
minuend, I-23.
mixed numbers, I-39.
money (U. S.), II-34; (British), II-34.
multiple, I-40.
multiplicand, I-19.
multiplication, I-17; tables, I-18, 19.
multiplier, I-19.
notation, I-5.
numeration, I-5.
numerator, I-37.
parallelogram, II-44.
percentage, I-55.
perpendicular, II-43.
prime factors, I-34.
principal (interest), II-17.
prisms, II-50; II-51.
powers, II-7.
product, I-17.
proportion, I-61; compound, II-56; inverse, II-54; practical, II-54.
quotient, I-26.
radical sign, II-10.
rate (interest), II-17.
ratio, I-61.
rational number, II-12
rectangle, II-42.
rectangular prisms, II-50.
remainder, I-23; I-23.
right angles, II-43.
Roman notation, I-10.
roots, II-10.

INDEX—Continued.

"rule of three," I-62; II-54.
significant figures, II-11.
simplifying, I-40.
solids, II-50.
sphere, surface of, II-44; volume, II-51.
square, II-7. See Rectangle.
square root, II-11.
subtrahend, I-23.
subtraction, I-23.
sum, I-11.
surds, II-12.
tens, I-6.
time, table of, II-34; in interest, II-17.
trapezium, II-44.
triangle, II-44.
units, I-6.
volume, II-33; II-49.
weights, II-32; avoirdupois, II-33; troy, II-33.